

Michel Hénon, a playfull and simplifying mind

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1 Introduction

Several chapters in this book present various aspects of Michel Hénon's scientific achievements that spread over a large range of subjects, and yet managed to make deep contributions to most of them. The authors of these chapters make a much better job at demonstrating the big advancements that Michel Hénon allowed in these fields than I could ever do. Here I rather present some facets of his personality that most appealed to me. Michel Hénon was a reserved person, almost shy, so it was not obvious for a young student to grasp the profoundness of his insight and what a marvelous advisor he could be. The two most prominent aspects of his mind, in my view, were his ability to simplify any scientific question to its core complexity, and to find the fun and amusing part in his everyday work, even in the tiniest details of his scientific investigations.

2 My first meeting with Michel Hénon

Michel Hénon was a teacher in the Diplôme d'Étude Approfondie (DEA, the equivalent of a Master at the time in France) *Turbulence et Systèmes Dynamiques*. Our first class with him was not on the study of dynamical systems, despite all his contributions to the field. Rather, it was a *practical* lecture on programming. Other teachers in this master were in charge of teaching us the theory and analytical study of dynamical systems. Since he professed to not be a good mathematician nor theorist, which none of us believed, he was concerned with numerical studies of dynamical system. Thus he felt that we needed to be taught basic good practice in programming to make sure we could the problem with the correct tools.

Michel Hénon was pragmatic and wanted to teach us useful knowledge. He could have decided to teach us an object-oriented language for software scientist that is very strict ... Instead, he decided to use the main language used in computational physics, Fortran, in its current incarnation Fortran77, and to give us very simple recipes to make our programmes clear, readable, correct

and resilient to basic errors such as typos. He relied on the good will of the programmers rather than on the grammar of the language and the compiler to write *good* programmes.

In Michel Hénon's view, one needs to have a structured mind to address any physical or mathematical question, and then one must have a structured view of the problem at hand. It followed that the tools used to solve the problem, in this case a computer programme, had to be well structured.

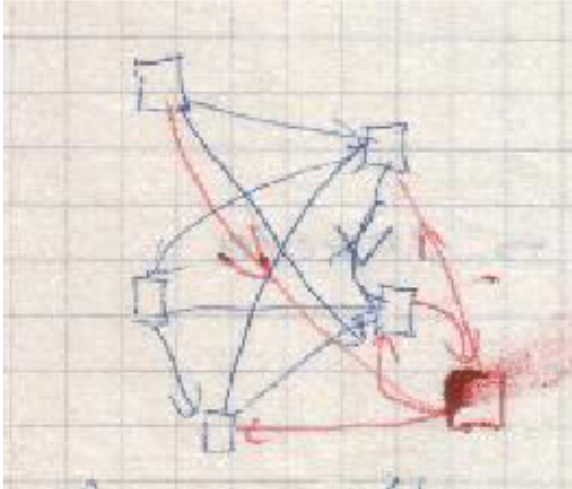
The main concern when writing a computer programme should be clarity, over computing time and memory resources. A good organisation is instrumental in avoiding errors from the start, thus limiting the possibility of uncaught errors at the end. If L is the length of the programme (say number of lines or statements), then the tuning time for a spaghetti-like programme (see Fig. 1) goes like L^2 , while that of a well structured programme is proportional to L . Since human time is more valuable than machine time, a programme has to be well structured. To achieve this, a programme should be organised using modules and sub-modules. As a bonus, it turns out that in most cases, this will allow the compiler to produce a more efficient binary code.

Structured programming is based on blocks. A block has a single input point and a single output point. Blocks can be composed together to form bigger blocks. One can define three different types of basic blocks: a sequence, a test or choice, and a loop. Obviously there are variants of these blocks. A test can have only two possible outcomes (say `if (x < 0) then {...} else {...}`) or a whole complement of possibilities (like in the `case ...` statement). A loop can have the completion test at the start (`do while (test) {...}`), at the end (`repeat {...} until (test)`) or in the middle (`mark {...} (test) {...} goto mark`). Equipped with these tools, Michel Hénon showed us how to use them on a simple example, counting the number of cycles in a bijection mapping on a group of 6 elements. For us, fresh master students who had barely had any programming class, this example was enlightening.

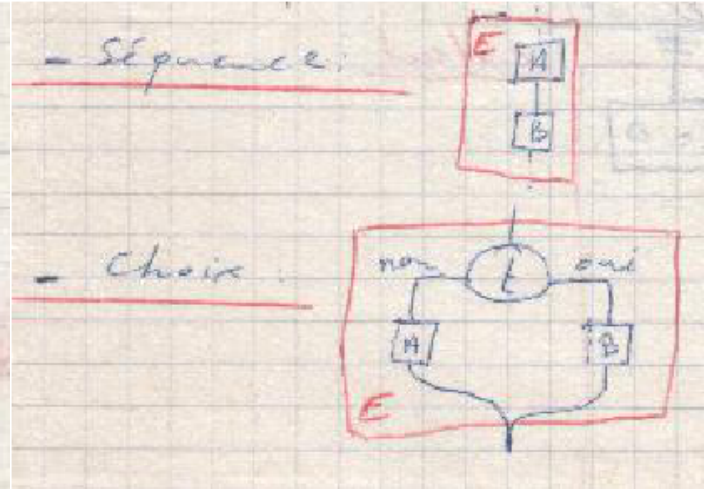
The structure of the programme can and should also be reflected in its actual appearance on the screen or listing. Within the limitations of the language grammar, one should choose variable and subroutine names that are meaningful. The text should be indented to reflect the structure of the logical blocks and modules. (Interestingly, this approach was pushed to the extreme in a very successful language, Python, where the scope of a block is given by indentation of the statement lines.) Because our mind gets a better grasp of what we see at once, one should limit the size of a module to the size of a page or screen, by using sub-modules when possible.

As important as the structure of the programme, one must document it. Comments should appear everywhere in the programme. They are of utmost importance for long term maintenance of the code. At the start of the code, one should write the details of the problem at hand, give the equations solved by the programme, give a list of all the variables (and always declare the type of the variable, whether this is mandatory for the programming language or not), and of the subroutine (modules) and their purpose. Together with this detailed description, one should also include the user's manual. Typical of his way of

(a)



(b)



(c)

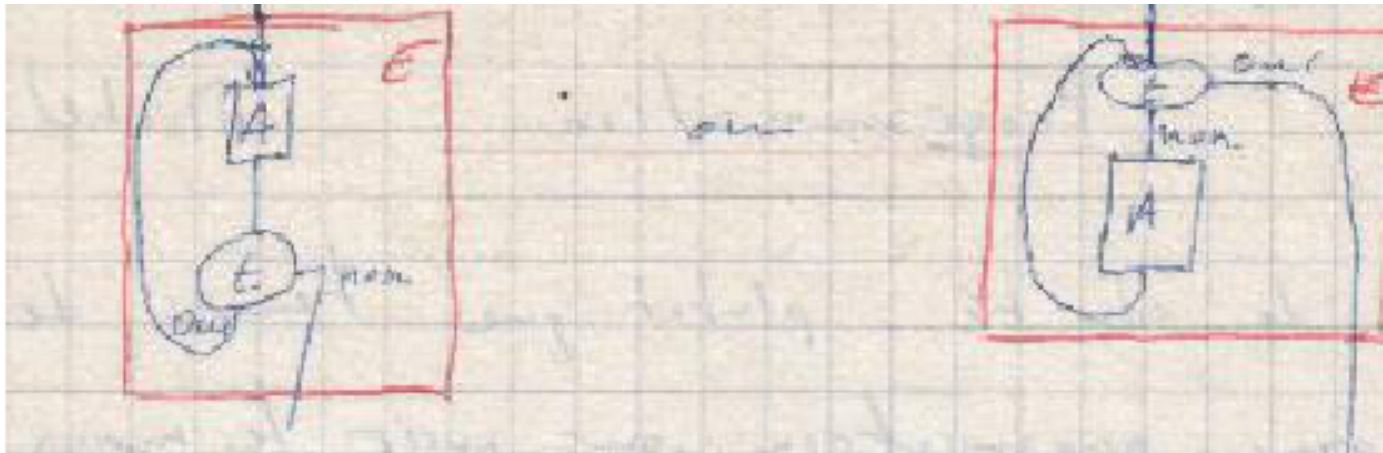


Figure 1: (a): the too frequent spaghetti plate type of programming, or mess, that should be avoided. Instead, programmes should be built from simple blocks. Only a small number of block types are needed to construct any programme, like sequences, tests (b), or loops (c). These graphs are copies of my personal notes at the time, which reproduced as best as I could the drawing from Michel Hénon on the black board.

doing things, Michel Hénon wrote a fortran programme that would take another fortran code with comments written in TeX format and produce a well written paper with comments forming the core text, and the computer code written verbatim in between.

Straight from the beginning, and through this simple class, Michel Hénon managed to convey a set of very important rules:

- Start with a global vision, apply a *top-down* approach;
- Extract the hard point of the problem;
- Perform a structured analysis of the problem;
- Write a clear documentation
- Apply a rigorous methodology.

The main strength of Michel Hénon was to stick to these rules by all means.

While teaching his class, Michel Hénon was calm, quiet and reserved. He really focussed on the essence of what he wanted to tell us. He avoided unnecessary complications aimed at showing how clever he was. This simplified and rigorous approach made a strong impression on the students and appealed very much to me. His very quiet style made him stand out amongst the teachers of the DEA.

3 Saturn's rings

I had to do a DEA reasearch project in spring 1983, hoping to continue on a 3rd cycle PhD thesis (the shorter format of a PhD thesis that was current at the time in France), at the time when we got the first Voyager data on Saturn's rings.

3.1 Context of our work

The rings of Saturn have an almost perfect circular symmetry; they are also extremely flat. Deviations of particle orbits from circular and coplanar shapes are of the order of 10^{-6} , with a few exceptions (eccentric rings, irregular rings, spokes). Therefore their spatial structure is essentially described by a single function: the *radial distribution*. Until that time, this distribution was believed to be also rather simple and smooth. The few structural details which could be seen from the Earth were considered as remnants of the formation process, or, as in the case of the Cassini division, were attributed to resonances with the major satellites.

The observations made by the Voyager probes have shattered this last belief and have revealed that the radial distribution is in fact extremely complex, with structure at all wavelengths down to the limit of resolution (Fig. 2). On the other hand, the circularity and flatness of the rings have been confirmed and even sharpened.

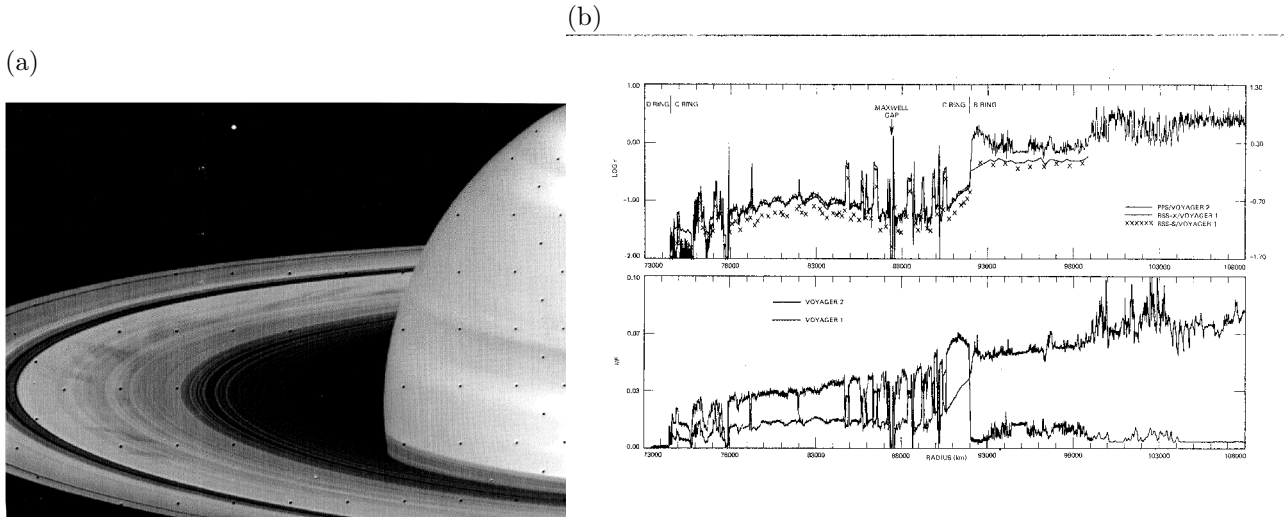


Figure 2: (a): One of the first photos of Saturn’s rings taken by the interplanetary probe VOyager 1. (b):

A major challenge for theorists was of course to explain these radial distributions. Essentially three kinds of explanations had been advanced: (i) resonances with external satellites; (ii) collective effects, leading to instabilities; (iii) cumulative effect of binary interactions. However, there were difficulties with each of these approaches, and at the time it is not clear which theory, or combination of theories, would ultimately provide the correct explanation.

Michel Hénon addressed the problem in his typical simplifying way (Petit & Hénon, 1987):

“In the present state of our knowledge, it seems premature to try to set up a fully realistic model of the rings... Therefore our objective will be, more modestly, to try to gain an understanding of some of the fundamental mechanisms at work. We will include in the model only some selected effects, and ignore the others. Thus, our ring models should be thought of as “model problems”. Our hope is that they will behave in some fundamental respects like real rings, and thus teach us something about ring physics.”

We thus constructed a *simplified* that retained the essence of the problem.

- We consider a 2-dimensional problem;
- The evolution of the system is a succession of binary interactions;
- Two physical effects of equal importance are included in the interactions: gravitation and inelastic collisions;

- Since we are interested in the radial structure, keep only the radial coordinate in the global evolution;

3.2 Satellite encounters and computer algebra

Once this course of action was defined, we first had to determine the effect of gravitational and collisional interaction between two particles in orbit around Saturn, at large distance. On the gravitational part, this is exactly the Hill's problem, first defined to study the motion of the Earth-Moon system around the Sun. Hill's equations are usually derived assuming a hierarchy of masses for the three bodies:

$$m_1 \gg m_2 \gg m_3, \quad (1)$$

and then proceeding in two steps: first take the limit $m_3 \rightarrow 0$, which gives the restricted three-body problem; then take the limit $m_2 \rightarrow 0$. Hill's problem is thus presented as a sub-case of the restricted three-body problem.

We showed that we can consider a more general situation: the ratio of the two masses m_2 and m_3 can be arbitrary; the only condition is the both masses should be small compared to m_1 :

$$m_1 \gg m_2, \quad m_1 \gg m_3. \quad (2)$$

So we fix the ratio m_2/m_3 and let both m_2 and m_3 tend to zero simultaneously. The equations obtained in this limit are identical to the classical Hill's equations, showing that (Hénon & Petit, 1986):

“The restricted problem is applicable to situations where one mass is much smaller than the two others; Hill's problem is applicable to situations where one mass is much larger than the two others.”

Once we have the Hill's equations of motion for the relative motion of the two satellites (masses m_2 and m_3), we must resort to numerical integration to find the motion. But this allows to know the motion only on a finite interval, where the gravitation between the two satellites plays a role. To determine a full solution from $t = -\infty$ to $t = +\infty$, we must develop analytic approximations, in the form of asymptotic series for the solution in the limit $t \rightarrow -\infty$ and $t \rightarrow +\infty$, i.e. when the satellites are far from each other. Let us call h the difference in initial semimajor-axis of the satellites before interaction, expressed in Hill's coordinates, and η the Hill's coordinate in the direction of relative motion at large distance. In the asymptotic expansion, the small parameter is η^{-1} . For initially circular orbits, we obtain series in powers of η^{-1} , with coefficients of η^{-i} for $i > 0$ involving powers of h ranging from h^{i-2} to h^{-2i+1} . This is cumbersome to derive, but still doable.

But when considering initially eccentric orbits, we have to deal with trigonometric series in the coefficients of the powers of η^{-1} , involving an angle θ and the relative eccentricity in Hill's coordinates, k . Michel Hénon could not resist the urge to use a computer to derive these formulae. One must remember that in the mid-1980's very little was available in terms of computer algebra, or even easily programmable computers. Michel Hénon had acquired a Do-It-Yourself

computer like a Zenith, and decided to use it to derive the series. The computer came with a very simple OS, a line editor, and a programming language, FORTH. He first wrote a small program to create a full-screen editor, because he felt writing a large program with a line editor was not convenient. Once this was available, he wrote a computer algebra program that could do power series expansions and trigonometric series expansions. With this tool at hand, he addressed our asymptotic expansion. One must realise that the full set of tools, and all the expansion coefficients had to fit inside the 32 KB of memory available in Michel Hénon's computer. The output of this program is shown in Fig. 3. Although we were confident the program worked, we still decided to check it. So each of us independently verified the expansion up to order 4. The program was right from the beginning, and we finally agreed with it. The expansions are of the form:

$$\begin{aligned}
\xi = & h + k \cos \theta - \frac{4}{3} s h^{-1} \eta_c^{-1} + \left(-\frac{8}{9} h^{-3} + \frac{7}{6} s h^{-1} k \sin \theta \right) \eta_c^{-2} \\
& + \left(\frac{17}{3} s h - \frac{32}{27} s h^{-5} - \frac{7}{3} s h^{-1} k^2 + \frac{23}{4} s k \cos \theta + \frac{28}{27} h^{-3} k \sin \theta \right) \eta_c^{-3} \\
& + \left[-\frac{44}{9} h^{-1} - \frac{160}{81} h^{-7} - \frac{14}{9} h^{-3} k^2 - \frac{49}{72} h^{-2} k \cos \theta \right. \\
& \left. + \left(-\frac{473}{16} h k + \frac{14}{9} h^{-5} k + \frac{99}{32} h^{-1} k^3 \right) \sin \theta - \frac{5}{4} s k^2 \sin 2\theta \right] \eta_c^{-4} + O(\eta_c^{-5}),
\end{aligned} \tag{3}$$

where $s = \text{sign}(\eta)$. The other position and velocity coordinates have similar expansions.

4 The inclined billiard

With the previous ingredients, we studied extensively the one-parameter family of orbits obtained by varying h for initially circular orbits (family of Satellite Encounters or SE) (Petit & Hénon, 1986). This family was found to be of amazing complexity; in fact it seems to possess the inexhaustible richness of details which is characteristic of nonintegrable problems in general.

Figs. 4a and b, taken from a collection of several hundred pictures, represent the relative motion (ξ, η) of the two satellites in Hill's coordinates. For their description, it will be convenient to think of the special case $m_2 \gg m_3$, and to identify the origin of the (ξ, η) with the satellite M_2 ; the curves represent then simply the motion of satellite M_3 .

Three successive phases can be distinguished in a typical orbit: (i) *approach* of the two satellites; (ii) *interplay*, or *temporary capture*: the two satellites remain close to each other (their distance is of order 1 in Hill's coordinates) and they perform complex relative motions; (iii) *departure*: the two satellites move away from each other. It can be shown that *permanent capture is possible only for a set of initial conditions of measure zero*.

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Ordre 5
xsi = + 1 - 4/3 e k - 8/9 et2 kt2 + 17/3 et3 k - 32/27 et3 kt3 - 44/9 et4 kt2
- 160/81 et4 kt4 - 152 et5 k - 40/27 et5 kt3 - 896/243 et5 kt5
u = - 2 et2 k + 51/2 et4 k - 78 et5 kt2 - 1140 et6 k
v = - 3/2 + 2 e k + 4/3 et2 kt2 - 11 et3 k + 16/9 et3 kt3 + 32/3 et4 kt2
+ 80/27 et4 kt4 + 1215/4 et5 k + 40/9 et5 kt3 + 448/81 et5 kt5
w = - 5 et3 k + 20/3 et4 kt2 + 303/2 et5 k + 40/9 et5 kt3 - 2464/3 et6 kt2
+ 160/27 et6 kt4
z = + 1 - 3/2 et2 + 4 et3 k + 15/8 et4 - 27 et5 k

Ordre 6
xsi = + 1 - 4/3 e k - 8/9 et2 kt2 + 17/3 et3 k - 32/27 et3 kt3 - 44/9 et4 kt2
- 160/81 et4 kt4 - 152 et5 k - 40/27 et5 kt3 - 896/243 et5 kt5
+ 2338/3 et6 kt2 + 32/81 et6 kt4 - 1792/243 et6 kt6
u = - 2 et2 k + 51/2 et4 k - 78 et5 kt2 - 1140 et6 k + 9126 et7 kt2
v = - 3/2 + 2 e k + 4/3 et2 kt2 - 11 et3 k + 16/9 et3 kt3 + 32/3 et4 kt2
+ 80/27 et4 kt4 + 1215/4 et5 k + 40/9 et5 kt3 + 448/81 et5 kt5
- 4739/3 et6 kt2 + 64/27 et6 kt4 + 896/81 et6 kt6
w = - 5 et3 k + 20/3 et4 kt2 + 303/2 et5 k + 40/9 et5 kt3 - 2464/3 et6 kt2
+ 160/27 et6 kt4 - 82065/8 et7 k + 6100/9 et7 kt3 + 800/81 et7 kt5
z = + 1 - 3/2 et2 + 4 et3 k + 15/8 et4 - 27 et5 k - 35/16 et6 + 152/3 et6 kt2

Ordre 7
xsi = + 1 - 4/3 e k - 8/9 et2 kt2 + 17/3 et3 k - 32/27 et3 kt3 - 44/9 et4 kt2
- 160/81 et4 kt4 - 152 et5 k - 40/27 et5 kt3 - 896/243 et5 kt5
+ 2338/3 et6 kt2 + 32/81 et6 kt4 - 1792/243 et6 kt6 + 246205/24 et7 k
- 592 et7 kt3 + 1120/243 et7 kt5 - 11264/729 et7 kt7
u = - 2 et2 k + 51/2 et4 k - 78 et5 kt2 - 1140 et6 k + 9126 et7 kt2

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Figure 3: Printer output from Michel Hénou's computer algebra program applied to the Hill' equation asymptotic expansions. Order 5, 6 and part of order 7 in η^{-1} are displayed.

The departure is asymptotically described by (3), with h replaced by h' for the final value, and two cases can be distinguished: (i) if $h > 0$, then $\eta \rightarrow -\infty$, while ξ remains finite and oscillates around a positive mean value; (ii) if $h < 0$, then $\eta \rightarrow +\infty$, and ξ oscillates around a negative mean value. When h varies, the orbit alternates from one kind of departure to the other.

For large values of h , the orbit of M_3 is only slightly perturbed. As h diminishes, the perturbation increases (Fig. 4, $h = 1.9$ and a loop appears ($h = 1.75$)). The shape of the orbit begins to change rapidly with h . Between $h = 1.7188$ and $h = 1.7164$ approximately, the orbit undergoes a series of complex changes of shape. This is the first transition zone (zone I in Fig. 4). From $h = 1.7164$ to $h = 1.6664$ approximately, things quiet down and the evolution of the family can again be followed: the shape of the orbit changes continuously and comparatively slowly with h . Then a new interval of violent changes begins, between $h = 1.6664$ and $h = 1.6497$. This is the second transition zone (zone II in Fig. 4). More transition zones occur when h diminishes to zero, separated by quiet intervals.

It should be noted that there is nothing absolute about the limits of the transition zones, as described above, nor even about their number. When descending to a finer level of details, one finds that each of the transition zones is resolved into several thinner transition regions, separated by quiet regions.

The net effect of the encounter is essentially characterized by the change in the final impact parameter h' . The transition zones correspond to intervals where h' changes abruptly from positive to negative value and back with small changes of h . Since $|h'| \geq |h|$, transitions imply a discontinuity in the family of orbits. This is puzzling since the differential equations that govern the motion contain no true singularities. Therefore the position of M_3 after a given time should be a continuous function of its initial position and velocity. To achieve a transition, we must pass through an orbit for which the duration of the “temporary capture” is infinite. This is achieved when *the orbit tends asymptotically toward a periodic orbit*. This is confirmed numerically. Fig. 5 represents the orbit for $h = 1.718779940$ which is the first transition encountered when coming from high values of h .

To understand what happens, we introduce the surface of section defined by $\eta = 0$, $\dot{\xi} > 0$: for each crossing of an orbit with the ξ axis in the positive direction, we plot a point with coordinate $\xi, \dot{\xi}$ (Fig. 6). An orbit is represented by a sequence of points. For a given value of the Jacobi constant Γ , a point in the surface of section defines completely the corresponding orbit. In particular, the next intersection point can be found. This defines the *Poincaré map* of the surface of section onto itself.

In the particular Poincaré map corresponding to the value of Γ for the orbit in Fig. 5, the periodic orbit corresponds to the fixed point P (Fig. 6). This orbit is unstable since it admits an asymptotic orbit. It has two real eigenvalues $\lambda_1 \simeq 1/640$ and $\lambda_2 \simeq 640$. An incoming orbit, associated with λ_1 , is represented by an infinite sequence of points Y_0, Y_1, Y_2, \dots , which lie on the *stable invariant manifold* of P converging exponentially on P (Fig. 6).

An outgoing orbit, associated with λ_2 , corresponds to a sequence of points

$\dots, Z_{-2}, Z_{-1}, Z_0$ lying on the *unstable invariant manifold* and diverging exponentially from P .

The fixed large values of the eigenvalues of the asymptotic periodic orbits responsible for the transitions preclude a detailed study of this phenomenon. To study the transition phenomenon in more detail, Michel Hénon developed a *model problem* which exhibits basically the same phenomenon and which is easier to study. With his usual very sharp insight, he decided to consider a problem that can be reduced to an explicit mapping with smaller and adjustable eigenvalues.

He thus defined the inclined billiard as follows (Hénon, 1988). A point particle moves in the (X, Y) plane. It bounces elastically on two fixed disks with radius r and with their centres in $(-1, -r)$ and $(1, -r)$. In addition, it is being subjected to a constant acceleration g which pulls it in the negative Y direction. Obviously, for most initial conditions, the particle will after a finite number of rebounds "fall" downwards, never to return. This is the equivalent of the separation of the two bodies in Hill's problem. To simplify the numerical computation of the mapping, Michel Hénon considered the *large r limit* which replaces the determination of the intersection of a parabola and a circle with that of two parabolas with parallel asymptotic directions. In this way, the full orbit can be computed analytically.

He showed that the various parameters defining the problem (total energy E , radius of the disks), can be reduced to a single dimensionless parameter Φ related to E and r .

He then defined a one-parameter family of orbits as in Hill's case, by assuming that the particle is initially at rest at a position (h, Y_o) , where Y_o is a positive constant and h is variable. This family of h -orbits exhibit the same kind of structure as Hill's problem. We find intervals of continuity, in which the orbit changes continuously, and transitions. For $h = -1$, for instance, the particle bounces indefinitely on the left disk in a straight vertical line; this is a periodic orbit, which is obviously unstable. $h = -1$ is a transition value, which separates two quite different kinds of motion: for $h < -1$, the particle falls toward the left and never returns, while for $h > -1$ it moves to the right and complex interplays involving the two disks are possible. A similar periodic orbit exists for $h = +1$. More generally, we may expect a transition for any solution which is asymptotic to one of these periodic orbits (Fig. 7).

Thanks to its careful design, Michel Hénon was able to derive a symbolic representation of the h -orbits. To a given h -orbit, he associated a sequence of binary digits

$$D : d_1, d_2, \dots, \quad (4)$$

with

$$d_j = \begin{cases} 0 & \text{if the } j^{\text{th}} \text{ rebound is on the left disk,} \\ 1 & \text{if the } j^{\text{th}} \text{ rebound is on the right disk.} \end{cases} \quad (5)$$

From D , he defined a number A by its binary representation:

$$A = 0.d_1d_2d_3\cdots = \sum_{j=1}^{\infty} 2^{-j}d_j. \quad (6)$$

He then proceeded to show that there is a close relation between the behaviour of the h -orbits, D and A , as given in Fig. 8.

Furthermore, for any given non-round A there corresponds exactly one h -orbit:

$$h = (\exp \Phi - 1) \sum_{j=1}^{\infty} 2^{-j\Phi} S_j, \quad (7)$$

where $s_j = -1$ if the j^{th} rebound is on the left disk, and $s_j = +1$ if the j^{th} rebound is on the right disk.

In his usual way, Michel Hénon so an intriguing behaviour in a physical problem of interest, found it amusing, and decided to study this behaviour for its own right. He devised a model problem that allowed him to study the phenomenon in its tiniest detail.

5 Collisions and fragmentation

The formation of asteroid families is the consequence of catastrophic impacts on former parent bodies. Many workers have addressed the question of high velocity collisions using numerical simulations taking into account both experimental and theoretical results. But to reproduce the puzzling steep size distributions of the asteroid families known at the time had been a task in which these modelling techniques of fragmentation have typically failed.

In the late 1990's . Campo Bagatin and I addressed the problem from the point of view of *geometrical constraints* (Campo Bagatin & Petit, 2001). The idea that geometrical constraints may play a role in the production of fragments was loosely found in the literature since the 30's. We decided to address the problem in a detailed and coherent way for the first time. Geometrical constraints stipulate that fragments cannot overlap, and that putting together all the fragments must reconstruct the parent body exactly.

Our main contribution to this study was to allow for arbitrarily given shape for the largest fragment, with the other fragments being either triaxial ellipsoids with approximate axis ratios, or bodies of unconstrained shape. The main goal here was to try and explain the observed steep slopes of the cumulative size distributions of known asteroid families. The geometric effects were considered in realistic ways, and size distributions of the produced debris were therefore obtained.

Running simulations with various numbers of fragments and sizes of the largest remnant, we observed size distributions alike the ones observed for the asteroid families. At the large-size end, the distributions exhibit a gap – the bigger the larger the first remnant – then a steep rise, followed by a power-law

regime and finally a plateau due to the finite number of fragments (Fig. 9). In the asteroid families, the (few) largest fragment(s) seem to have a size and shape that is (are) stochastic, essentially related to the past history of the parent body, and to the impact energy for the size of the largest one. then we see a steep rise and a power-law like regime. The power-law regime in our simulations was very satisfying, but it was unclear where it came from, and even worse, we did not see how to determine the exponent, except by measuring it on our results.

At the time, A. Campo Bagatin and I were working at Nice Observatory, in the same corridor as Michel Hénon. Every day, we had lunch all together at the rather famous Nice Observatory restaurant, enjoying its very fine cuisine while overseeing Nice and the *Baie des Anges*. During the lunch, I frequently had discussions with Michel Hénon about many things, including our own work. So one day I mentioned to him our findings in the geometrically constrained fragmentation simulations, in particular the appearance of a power-law regime. He listened to me carefully, as he always did, and then we talked about other things, and returned to our offices, each minding his own work, or so I thought.

A few days later, he showed up in my office, and gave me a little piece of paper which I still keep dearly, and which is reproduced in Fig. 10. Since I only explained verbally the problem and did not give him any of our results, his solution had to come from some analytical understanding of the process, not a measurement of the exponent from a plot. And he could not resist the pleasure to exite my curiosity. Of course, he was right, as can be seen from the dashed-line in Fig. 9. When pressed to explain himself, he gave me the outline of the reasoning, which not only gave the answer we were looking for, but also gave a general formula expressing the exponent α as a function of the dimensionality a of the problem:

$$\alpha = \frac{a^2 + 2a - 1}{a + 2} \quad (8)$$

which yield $\alpha = 2.8$ for $a = 3$.

This result was very important for our work on geometrical constraints in fragmentation simulations, as it gave us a full understanding of the process. But unfortunately, it showed that the size distribution for the smallest fragments (smaller than the thousand largest) is completely determined by the algorithm, and hence bears no physical significance. We therefore had to restrict our study to the thousand largest fragments, and look at the effect of the size and shape of the largest fragment: cratering case, spallation, ellipsoidal core, conic antipodal fragment, ...

6 *Le mot de la fin*

Michel Hénon was certainly the best possible advisor I could dream of. He gave me the rigorous training I needed to embrace a research career. He reinforced in my head the feeling that one must have a rigorous approach to research, as to any other undertaking, trying to clearly define one's goal and context, and consider all consequences of one's hypotheses, and also not be lured into

publishing before you have done a carefull and scientifically interesting work. Even more important in some respects, he showed my that one must find the fun in one's work, so as to keep interested and efficient at all times. If one thing, this rigorous approach is what I try to apply to myself every day in my work, and try to convey to my students.

Michel, You will stay in my mind and my heart for ever.

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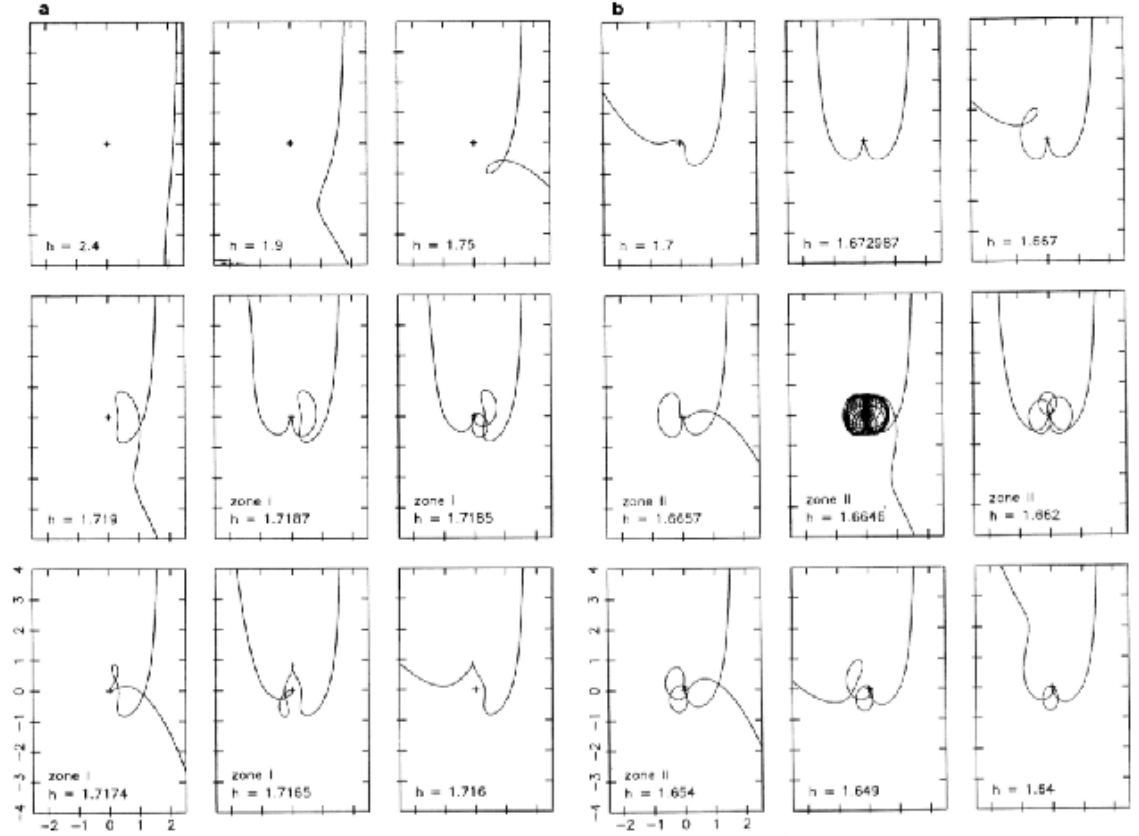


Figure 4: (a) Beginning of family SE. Each frame corresponds to one particular value of the family parameter h . The curve represents the relative motion of one satellite with respect to the other, in Hill's coordinates (ξ in abscissa, η in ordinate). The initial approach is downward from $\eta = +\infty$, in the first quadrant. The orbit belonging to transition zones are labeled zone I, zone II, ... (b) Continuation of family SE.

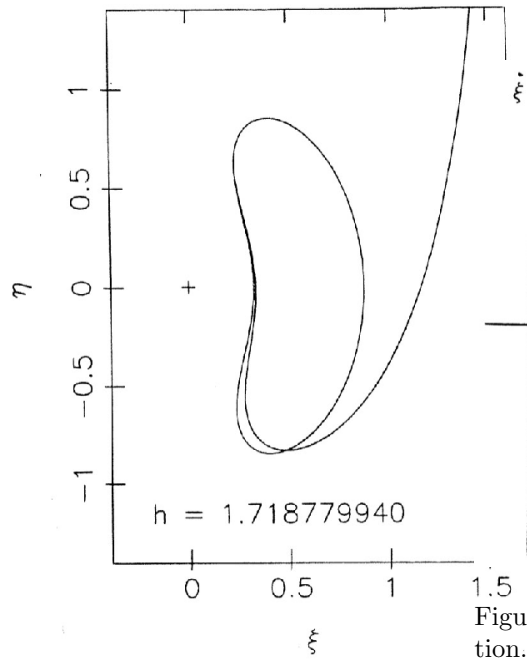


Figure 5: An orbit of family SE which is asymptotic to an unstable periodic orbit.

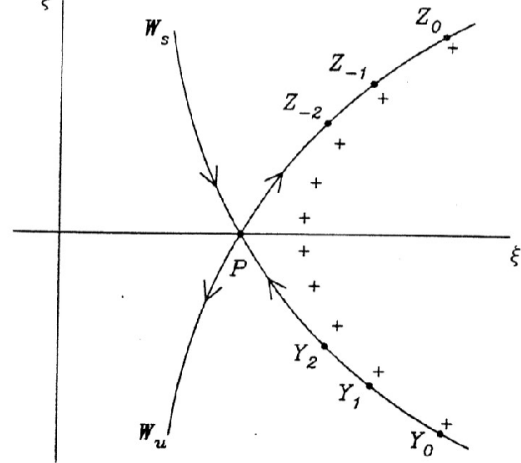


Figure 6: Sketch of the surface of section. The value of λ_1 has been artificially increased to show the structure more clearly.

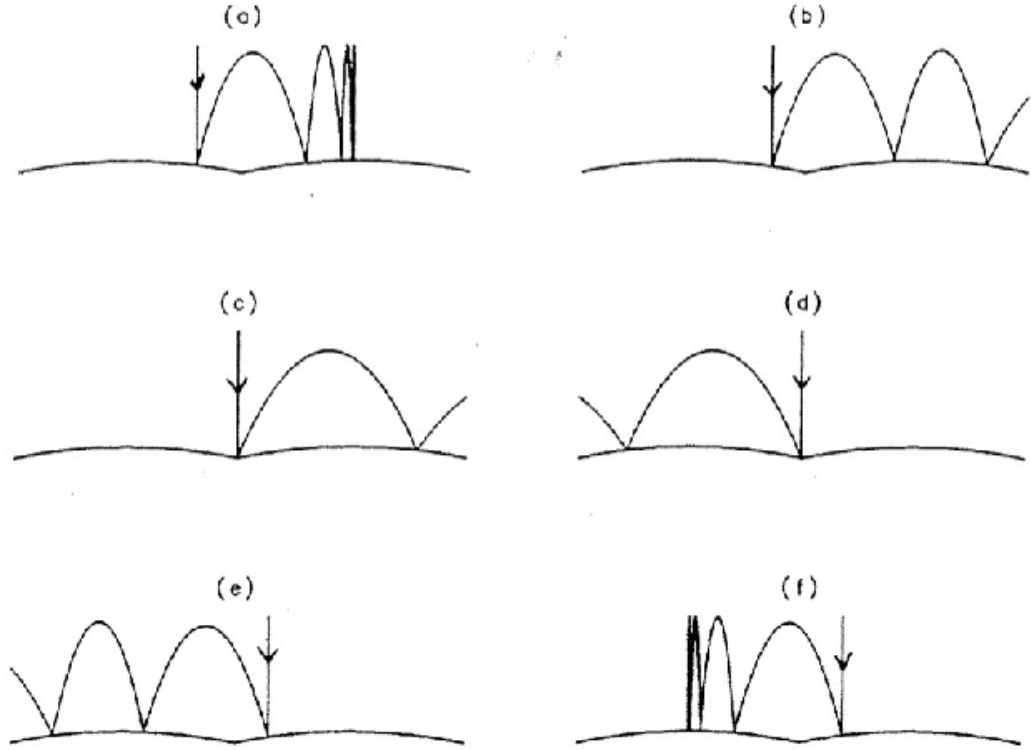


Figure 7: Selection of simple members of the h -orbit family: (a) right-asymptotic orbit; (b), (c) right-escaping orbits ; (d), (e) left-escaping orbits; (f) left-asymptotic orbit.

<u>orbit</u>	<u>D sequence</u>	<u>A</u>
right-escaping	1-ending	round
right-asymptotic	1-ending	round
left-escaping	0-ending	round
left-asymptotic	0-ending	round
oscillating	oscillating	non-round

Figure 8: Correspondance between the type of orbit, the D sequence and the number A .

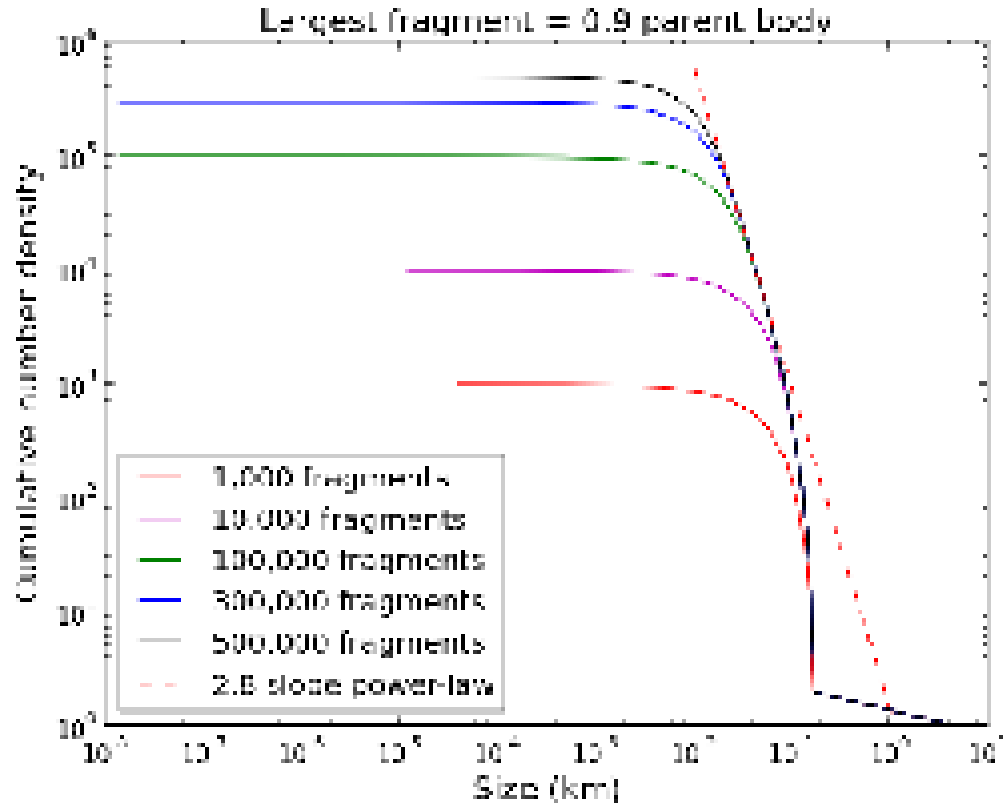


Figure 9: Size distribution of spherical fragments with a largest remnant 0.9 times the mass of the parent body. Different curves correspond to different total numbers of fragments simulated.

exposant = 2.8 ??

Figure 10: Scanned version of the little piece of paper Michel Hénon brought to my office one day, giving the answer to the geometrically constrained fragmentation simulations.